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An analogy between the Compton effect and reflection from a moving mirror

D G Ashworth
Electronics Laboratories, University of Kent, Canterbury, Kent, UK

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Abstract. It is shown that the Compton effect can be simulated if the scattering electron is replaced by a perfectly reflecting mirror which moves with a constant velocity of magnitude such that the initial and final electron velocities are equal in magnitude but opposite in direction as seen from the frame of reference in which the mirror is stationary.

The interaction of a photon with a free electron, as depicted in figure 1, was first described by Compton (1923). In the laboratory frame, S, a photon of initial energy $E = h\nu$, is in collision with a stationary electron. The photon is deflected through an angle ϕ and has an energy $E' = h\nu'$ after collision. The electron, at rest prior to the collision with energy $E_0 = m_0c^2$, moves off after the collision with velocity V and energy $E = mc^2$ at an angle ψ with respect to the initial direction of propagation of the photon. The symbols ν and ν' refer to the frequency of the photon before and after collision respectively, m_0 is the rest mass of the electron,

$$m = m_0\left(1-\frac{V^2}{c^2}\right)^{-1/2}$$

is the mass of the electron when it is travelling at velocity V, c is the velocity of electromagnetic waves in free space and h is Planck's constant.

Applying the law of conservation of energy gives

$$h\nu + m_0c^2 = h\nu' + mc^2 \tag{1}$$

and the law of conservation of momentum gives

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\phi + mV\cos\psi \tag{2}$$

and

$$0 = \frac{h\nu'}{c}\sin\phi - mV\sin\psi. \tag{3}$$

It has been shown previously by Ashworth and Jennison (1974) that equations (1)–(3) combine together to give

$$\frac{\nu'}{\nu} = \frac{1-(V/c)\cos\psi}{[1-(V^2/c^2)]^{1/2}} \tag{4}$$

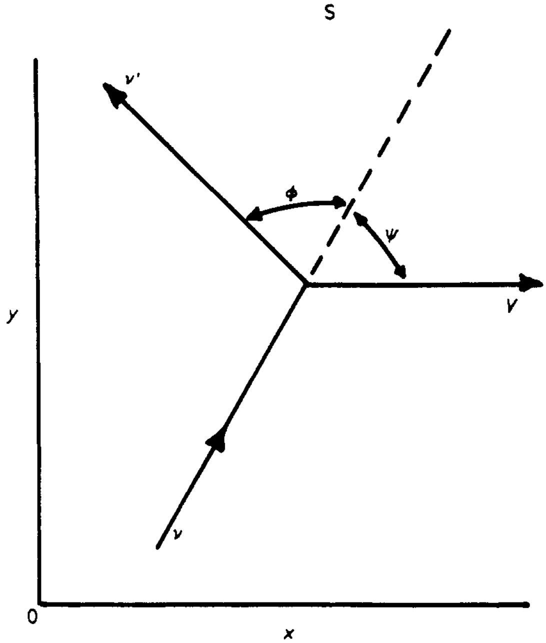

Figure 1. The Compton effect in the laboratory frame, S.

and

$$\cos(\phi+\psi)=-\frac{\cos\psi-(V/c)}{1-(V/c)\cos\psi}. \tag{5}$$

Let us now consider the relativistic Doppler equations applicable to the reflection of light incident at any angle upon a perfectly reflecting mirror moving with constant velocity in any arbitrary direction. Figure 2 is drawn in the inertial frame, S, of the laboratory in which a mirror is moving with constant velocity v parallel to the x axis. The normal to the plane of the mirror is at an angle α with respect to the direction of motion of the mirror and ν_1 and ν_2 are the frequencies of the incident and reflected light respectively. The angles of the light rays are those between the positive direction of the x axis and the positive direction of motion of the light ray. The angles of incidence and reflection, thus defined, are denoted by ϕ_1 and ϕ_2 respectively. It has previously been shown by Ashworth and Davies (1976), that

$$\frac{\nu_2}{\nu_1}=\frac{[\tan\alpha+(v/c)\sin\phi_1]^2+[1-(v/c)\cos\phi_1]^2}{1-(v^2/c^2)+\tan^2\alpha} \tag{6}$$

and

$$\cos\phi_2=\frac{(\cos\phi_1\tan\alpha+\sin\phi_1)^2-[1-(v/c)\cos\phi_1]^2}{\cos\phi_1\{[\tan\alpha+(v/c)\sin\phi_1]^2+[1-(v/c)\cos\phi_1]^2\}}. \tag{7}$$

If we now consider the possibility that the Compton effect might be an example of light being reflected from a perfectly reflecting moving mirror then

$$\frac{\nu'}{\nu}\equiv\frac{\nu_2}{\nu_1}, \qquad (\phi+\psi)\equiv\phi_2, \qquad \psi\equiv\phi_1. \tag{8}$$

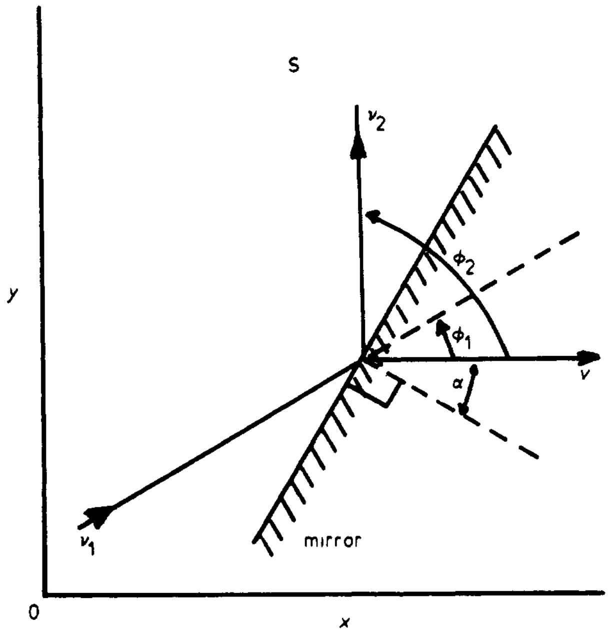

Figure 2. Reflection of an electromagnetic wave from a mirror which is moving with velocity v parallel to the x axis of the laboratory frame, S.

Because of equation (8) we can equate equations (4) and (6). Similarly we can equate equations (5) and (7), thus obtaining two simultaneous equations. After lengthy reduction these simultaneous equations can be shown to have as a possible solution,

$$v=\frac{c^2}{V}\left[1-\left(1-\frac{V^2}{c^2}\right)^{1/2}\right] \tag{9}$$

(cf equation (3) of Jennison 1978) and, either

$$\tan\alpha=\frac{2(c/V)\sin\psi[1-(V^2/c^2)]^{1/2}\{[1-(V^2/c^2)^{1/2}]\cos\psi-[\cos\psi-(V/c)]\}}{(V/c)-2\cos\psi+(V/c)\cos^2\psi} \tag{10}$$

or

$$\tan\alpha=0. \tag{11}$$

The scattering electron in the Compton effect is therefore acting like a perfectly reflecting mirror moving at the constant velocity v given by equation (9) and this velocity is independent of the angles of incidence and reflection of the radiation. Equation (9) can be rewritten in the form

$$V=\frac{2v}{1+(v^2/c^2)} \tag{12}$$

which, by the velocity addition formula, means that an observer who is stationary with respect to the moving mirror during the reflection process, in the frame S′ say, would ‘see’ the electron moving in the $+x'$ direction with velocity v after scattering had taken place. The same observer would have ‘seen’ the same electron moving in the $-x'$

direction with velocity v before scattering had taken place. This observer could therefore interpret these observations to mean that the 'scattering' takes place from a mirror moving at the *mean* velocity for the *whole* interaction as 'seen' in S′.

Using equation (12) enables equation (10) to be written in the form

$$\tan\alpha = -\left(1-\frac{v^2}{c^2}\right)\sin\psi\left(\cos\psi-\frac{v}{c}\right)^{-1}. \tag{13}$$

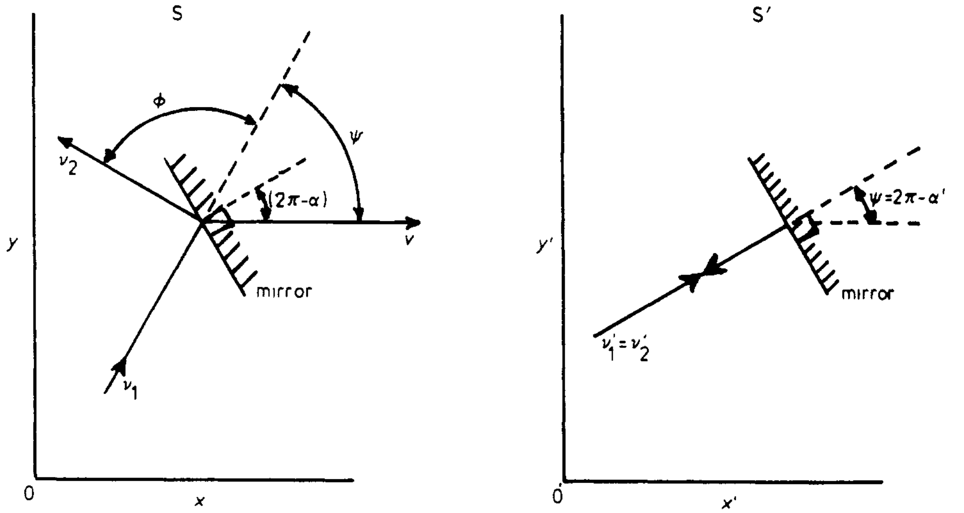

Figure 3. Simulation of the Compton effect by a perfectly reflecting mirror which satisfies the conditions given in equations (9) and (10). The simulation is shown in the laboratory frame, S, and in the frame S′ in which the mirror is stationary.

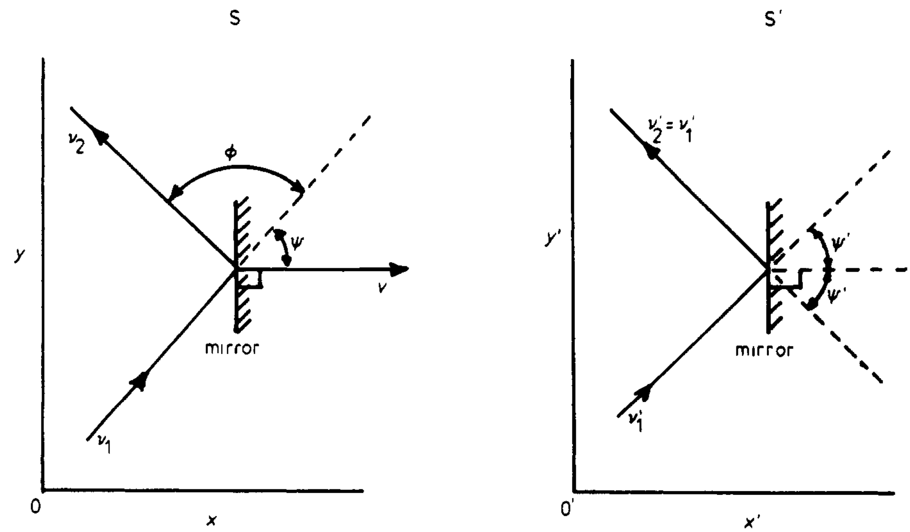

Figure 4. Simulation of the Compton effect by a perfectly reflecting mirror which satisfies the conditions given in equations (9) and (11). The simulation is shown in the laboratory frame, S, and in the frame S′ in which the mirror is stationary.

But, if primed letters refer to the frame S′, then

$$\tan \alpha = \left(1 - \frac{v^2}{c^2}\right)^{1/2} \tan \alpha'. \tag{14}$$

Using equation (14) in conjunction with equation (13) and the aberration equations gives

$$\tan \alpha' = -\tan \psi' \tag{15}$$

and, hence,

$$\phi_2' = \phi_1' + 180°. \tag{16}$$

Alternatively, if we use equation (11) then

$$\tan \alpha' = 0. \tag{17}$$

A perfectly reflecting moving mirror, therefore, has the same effect on a photon as that produced by the scattering electron in the Compton effect in the two cases depicted in figures 3 and 4.

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